Summary

First-order differential equations which govern the rotational motion of a triaxial rigid body under the influence of arbitrary external torques have been derived for two different Euler rotation sequences. The derivations are very straightforward, relying in no way on having a solution to the free-motion problem. The equations given here are suitable for numerical integration and offer advantages in speed and accuracy over the usual Euler's equations when the magnitude of the total external torque is small compared to the magnitude h of the rotational angular momentum $\mathbf{H}$; for they take full advantage of the fact that in such a case $\mathbf{H}$ is an almost constant vector. Since the solution to the problem of describing the rotational motion of a free, triaxial, rigid body with respect to a set of axes one of which lies along $\mathbf{H}$ is well-known,[4] the equations derived in this Note are also suitable for use in studying the effects of perturbing torques on the rotational motion of artificial satellites.[1,5]

References

[1] Crenshaw, J. W. and Fitzpatrick, P. M., "Gravity Effects on the Rotational Motion of a Uniaxial Artificial Satellite," *AIAA Journal*, Vol. 6, No. 11, Nov. 1968, pp. 2140–2145.

[2] Sterne, T. E., *An Introduction to Celestial Mechanics*, Interscience, New York, 1960, pp. 104–108.

[3] Deprit, A., "Free Rotation of a Rigid Body Studied in the Phase Plane," *The American Journal of Physics*, Vol. 35, No. 5, 1967, pp. 424–428.

[4] Whittaker, E. T., *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*, Cambridge University Press, 1965, pp. 144–148.

[5] Cochran, J. E., "On the Effects of Gravity-Gradient Torque on the Rotational Motion of a Triaxial Artificial Satellite," Dissertation, Aug. 1970, Univ. of Texas, Austin, Texas.

Shock-Wave Strength for Separation of a Laminar Boundary Layer at Transonic Speeds

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STEWARTSON and Williams[1] have derived asymptotic representations which describe the shock-induced separation of a supersonic laminar boundary layer. Their analysis expresses in a systematic way the idea, proposed in the same context by Lighthill,[2] that an abrupt change in pressure causes changes in the boundary-layer profile which are described approximately by inviscid-flow equations, except in a very thin viscous sublayer where the boundary-layer equations are retained in the first approximation. The results of Ref. 1 are not uniformly valid when the external-flow Mach number M_∞ is close to one, because linear supersonic theory has been used to relate the pressure and the flow deflection just outside the boundary layer. According to transonic small-disturbance theory, the changes in pressure are of the same order as the two-thirds power of the flow deflection angle. Separation at transonic speeds will occur if this pressure rise is sufficient to overcome the viscous stresses acting on a fluid element very close to the surface and to cause a change in its velocity which is of the same order as its velocity just upstream of the pressure rise. Hence, as in Ref. 1, near the surface (i.e., in a very thin sublayer) the momentum equation should express a balance among viscous, pressure, and inertia forces, and the term representing acceleration should be nonlinear. The preceding ideas form the basis for the analysis outlined below, which follows very closely the derivation described in detail for a related problem in Ref. 5.

Let $\bar{x}/L$ and $\bar{y}/L$ be coordinates along and normal to the surface, respectively, made nondimensional with a characteristic length L (e.g., chord length). Stretched coordinates are defined as follows:

$$x = R^{\alpha}\bar{x}/L, \quad y = R^{1/2}\bar{y}/L, \quad Y = R^{\beta}\bar{y}/L = R^{\beta-1/2}y \qquad (1)$$

where α and β are to be determined and R is the Reynolds number $\bar{\rho}_w u_\infty L/\bar{\mu}_w$ based on the freestream velocity and on thermodynamic properties at the surface. Throughout most of the boundary layer the flow will be described by small perturbations on an initial velocity profile $\bar{u}/\bar{u}_\infty = u_0(y)$. Since $u_0(y) = O(y)$ as $y \to 0$, the sublayer velocity will be $O(R^{1/2-\beta}Y)$ as $Y \to \infty$. Just outside the boundary layer (for $y \to \infty$ but $\bar{y}/L \to 0$) $p = O(\theta^{2/3})$, where p is the relative change in pressure from the value just upstream of the disturbance and θ is the flow deflection angle. It will be anticipated that $p_y \sim 0$ throughout the boundary layer, and the terms $\partial(\bar{\rho}\bar{u})/\partial\bar{x}$ and $\partial(\bar{\rho}\bar{v})/\partial\bar{y}$ appearing in the continuity equation will be required to be of the same order. These considerations suggest the following assumed asymptotic expansions:

$$\bar{u}/\bar{u}_\infty \sim u_0(y) + \lambda_1(R)u_1(x,y) + \dots \qquad (2a)$$

$$\bar{v}/\bar{u}_\infty \sim R^{\alpha-1/2}\lambda_1(R)v_1(x,y) + \dots \text{ (main b.l.)} \qquad (2b)$$

$$\bar{u}/\bar{u}_\infty \sim R^{-\beta+1/2}U_1(x,Y) + \dots \qquad (3a)$$

$$\bar{v}/\bar{u}_\infty \sim R^{\alpha-2\beta+1/2}V_1(x,Y) + \dots \text{ (sublayer)} \qquad (3b)$$

$$\gamma^{-1}p \sim \epsilon p_1(x) + \dots \qquad (4)$$

where γ = ratio of specific heats; $\lambda_1(R) \ll 1$; and $R^{\alpha-1/2}\lambda_1 = O(\epsilon^{3/2})$ for the transonic case. The first term in the density is $\bar{\rho}/\bar{\rho}_w \sim \rho_0(y)$, and in the sublayer $\bar{\rho}/\bar{\rho}_w \sim \rho_0(0) = 1$. Substitution in the x-momentum equation then gives, for the main part of the boundary layer and the sublayer, respectively,

$$\lambda_1 R^{\alpha}\rho_0(u_0 u_{1_x} + v_1 u_0') = -\epsilon R^{\alpha}p_1' + \dots \qquad (5)$$

$$R^{-2\beta+1+\alpha}(U_1 U_{1_x} + V_1 U_{1_Y}) = -\epsilon R^{\alpha}p_1' + R^{\beta-1/2}U_{1_{YY}} + \dots \qquad (6)$$

In the analogous derivation of Ref. 1, $R^{\alpha-1/2}\lambda_1 = O(\epsilon)$ and a formulation for $\alpha < \frac{1}{2}$ is sought because it is anticipated that the pressure change takes place over a distance large compared with $R^{-1/2}$. Therefore the pressure gradient is absent from the first approximation to Eq. (5), and the solution for v_1 has the form $v_1(x,y) = u_0(y)G_1'(x)$, implying that the flow deflection $\theta \sim R^{\alpha-1/2}\lambda_1 v_1/u_0$ is independent of y and is a consequence only of the displacement effect of the sublayer. Requiring that the flow deflection remain of the same order in the sublayer, so that $\lambda_1 R^{\alpha-1/2} = O(R^{\alpha-\beta})$, and equating the orders of the three terms in Eq. 6, one obtains the results given in Ref. 1: $\alpha = 3/8$, $\beta = 5/8$, $\epsilon = O(R^{-1/4})$, $\lambda_1 = O(R^{-1/8})$; the results for ϵ and for α were proposed originally by Lees[3] and by Lighthill,[2] respectively.

At transonic speeds, with $\lambda_1 R^{\alpha-1/2} = O(\epsilon^{3/2})$, the corresponding results are $\alpha = \frac{3}{10}$, $\beta = \frac{3}{5}$, $\epsilon = O(R^{-1/5})$, $\lambda_1 =$

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$O(R^{-1/10})$. This value of ϵ does not agree with the earlier result $\epsilon = O(R^{-2/5})$ proposed by Stewartson.[4] His derivation neglected displacement effects, and the external pressure distribution was prescribed to be the pressure distribution given by Taylor's solution for a weak shock wave. However, it can be shown that the pressure perturbation resulting from the outward streamline displacement in Stewartson's solution is of order $\epsilon^{1/6}$, much larger than the imposed pressure rise of order ϵ. This observation tends to support the present description in terms of a free interaction. The pressure rise of order $R^{-1/5}$ then presumably corresponds to the first appearance of separation. In transonic small-disturbance theory, the important similarity parameter can be interpreted as the ratio of $M_\infty^2 - 1$ to a typical pressure change. For the present case the corresponding similarity parameter may be written as $(M_\infty^2 - 1)/R^{-1/5}$.

As in the case of higher Mach numbers, increasing the shock strength would be expected to lengthen the region of separated flow and thus to increase the distance between the separation point and the main part of the pressure increase. Therefore one is led to the assumption that a shock strength of order $R^{-1/5}$ at transonic speeds also corresponds to the first appearance of shock bifurcation. For $R^{-1/5} \ll \epsilon \ll 1$ a pressure rise of order $R^{-1/5}$ accompanied by separation would be expected to occur at the base of the forward branch of a lambda shock, over a streamwise distance of order $R^{-3/10}$. Since the equations do not seem to permit a pressure rise of this order in any shorter region including the separation point, it appears that immediately outside the boundary layer the outgoing compression waves have not yet coalesced to form a shock. This conclusion also seems to be implied by the schlieren photographs and surface-pressure measurements of Refs. 6 and 7. The description of separation therefore could be carried out by numerical solution of the sublayer equations as in Ref. 1, with the transonic approximation relating p and θ to replace the linear-theory relation just outside the boundary layer. Since the pressure gradient associated with incoming waves is expected to be much smaller, the required condition is found by expanding the simple-wave solution for $M \to 1$; the result is $p \sim -\gamma(\gamma + 1)^{-1/3}(\frac{3}{2})^{2/3} (\theta + \nu_\infty)^{2/3} + \gamma(\gamma + 1)^{-1} (M_\infty^2 - 1)$, where $\nu_\infty = -(\frac{2}{3})(\gamma + 1)^{-1}(M_\infty^2 - 1)^{3/2}$. The distance downstream to the main branch of the lambda shock increases as ϵ increases, and the main branch is directly followed by an expansion.[6,7] The large rise in surface pressure therefore occurs slightly downstream of the main shock; if the boundary layer reattaches, the over-all flow pattern is similar in many respects to that observed at higher Mach numbers.

References

[1] Stewartson, K. and Williams, P. G., "Self-Induced Separation," *Proceedings of the Royal Society A*, Vol. 312, 1969, pp. 181–206.

[2] Lighthill, M. J., "On Boundary Layers and Upstream Influence. II. Supersonic Flows without Separation," *Proceedings of the Royal Society A*, Vol. 217, 1953, pp. 478–507.

[3] Lees, L., "Interaction Between the Laminar Boundary Layer over a Plane Surface and an Oblique Shock Wave," Rept. 143, 1949, Aeronautical Engineering Lab., Princeton Univ.

[4] Stewartson, K., "On the Interaction Between Shock Waves and Boundary Layers," *Proceedings of the Cambridge Philosophical Society*, Vol. 47, 1951, pp. 545–553.

[5] Messiter, A. F., "Boundary-Layer Flow near the Trailing Edge of a Flat Plate," *SIAM Journal of Applied Mathematics*, Vol. 18, 1970, pp. 241–257.

[6] Ackeret, J., Feldmann, F., and Rott, N., "Investigations of Compression Shocks and Boundary Layers in Gases Moving at High Speed," Rept. 10, 1946, Institut für Aerodynamik E.T.H., Zurich; translated as NACA TM 1113.

[7] Liepmann, H. W., "The Interaction Between Boundary Layer and Shock Waves in Transonic Flow," *Journal of the Aeronautical Sciences*, Vol. 13, 1946, pp. 623–637.

Nonequilibrium Dissociating Nitrogen Flow over a Circular Cylinder

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1. Introduction

WHEN the kinetic energy of a high-speed stream of diatomic gas is near the dissociation energy for the gas, a body placed in the stream can cause dissociation to occur in the stagnation region by virtue of the bow shock wave that thermalizes much of the kinetic energy. The dissociation rate can be characterized by the distance through which the gas has to move before reaching equilibrium. When the body size is similar to this characteristic dissociation distance, nonequilibrium effects have to be taken into account. Such flows are of interest in re-entry situations and have been studied extensively using various theoretical models.[1–4] Of particular interest, because of its practical relevance, is the flow of nitrogen over a blunt body. Until recently, nitrogen flows of sufficient speed and density could not be produced in the laboratory on a large enough scale to produce significant effects due to chemical reaction. Experimental studies of reacting hypersonic flows were restricted to gases with lower dissociation energies, such as oxygen (Spurk and Bartos[5]), or to free-flight ranges, in which flowfield measurements are difficult and expensive.

The recent completion of the large free-piston shock-tunnel at the Physics Department, A.N.U. made it possible to study reacting nitrogen flows in the laboratory, (Stalker and Hornung[6]). The first results of measurements in this tunnel, which were made using as a model a circular cylinder in cross flow, are described here, and compared with theoretical calculations. While this facility is capable of more than twice the flow speed of the present experiment, the conditions chosen give a reacting flow of good quality, with low freestream dissociation.

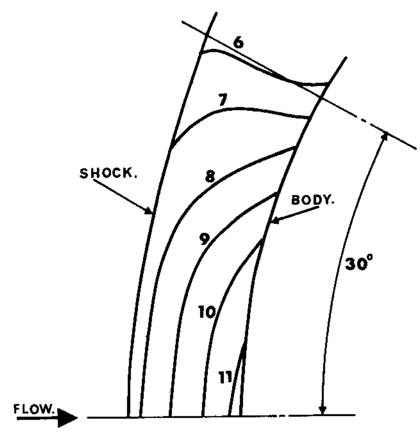

Fig. 1 Contours of density in reacting nitrogen flow over a 6-in.-diam circular cylinder. Calculation based on Freeman's model, at conditions of experiment. The numbers on the contours represent the density as multiples of freestream density.

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